PRICING BERMUDAN-TYPE CALL OPTION THROUGH BINOMIAL TREE METHOD

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Abstract

Bermudan option is a type of option that has characteristics between American option and European option whose its value never exceeds the value of the American option and is never less than the European option. The objective of this research is to calculate Bermudan call option of John Keels Stock through the binomial tree method using statistics software of Matlab R2010a. Assessment of Bermudan type option relates to discrete issues, in which the Bermudan type option has a certain number of times of early exercise specified in the option contract, where such times can only be made at some time prior to the option due date. Precise pricing for Bermudan type option can be obtained by discrete models such as the binomial tree method, a numerical method that is one of the most popular approaches for calculating option prices. This research uses time series data obtained from BNI Financial Update Corner, FEB UGM. The Bermudan call option price calculation will be compared with the calculation of European option pricing and American option price with underlying asset without dividend. The results show that the price of John Keels's Bermudan type call option using the binomial tree method yields the same value as American type call option and European type call option.

Keywords: Bermudan Type Option, Binomial Tree Method, Matlab R2010a, Spss 20

1. INTRODUCTION

Option is one of the financial derivative products are very popular give the right but not the obligation, to perform at or prior to maturity at a predetermined price. Options become popular because of the price of the option offered on the market are relatively cheap compared to the price or value of the stock, so stock investors can invest with a smaller capital compared to investing directly in the stock price or the actual value.

Options are many sells in the market generally is the American-type option on the grounds of flexibility that can be done before the implementation of the option period runs out or known by the feature early exercise. One type of option to allow exercise early is a Bermudan option. Bermudan option is a type of non-standard American option that has a number of limited time only on certain dates are allowed to exercise early, as has been stated in the contract. Contrary to the American option where feature early exercise can be used until maturity option, Bermudan can be run only on a number of discrete predetermined time.

Rate Bermudan option associated with discrete issues where it is generally recognized that there is no complete solution and must be solved numerically. Three financial economist, John Cox, Stephen Ross, and Mark introduces discrete models such as binomial tree model to solve the problem of complex

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option pricing model (1979). According to Hull (2002), binomial tree is a simple and powerful technique that can be used to solve many problems option pricing.

2. LITERATURE REVIEW

Bermudan option is a combination of both the character of the option, the option is European and American options. Adapting the European option properties, Bermudan option can only be exercised at the times specified in the contract and at the time of maturity. But not completely the same as the European option, this option allows the holder to exercise early such American options. American options can be implemented at any time, while the Bermudan option has a limited number of dates in which the option can be exercised annually, quarterly or monthly (Schweizer, 2012).

Binomial Distribution

The binomial distribution is a discrete probability distribution of the number of successes in n trials yes / no (successful / failed) that are free. Density function of the binomial distribution is

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}, x = 0,1,2, \dots, n$$

p is the probability of success, q is an opportunity to fail, that is (1 - p), x is the number of successes in the n repetitions and n is the number of repetitions.

Binomial Tree Model Stock Price

Cox, Ross & Rubinstein (1979), assuming the stock price follows the multiplicative binomial process during discrete time periods. An option with maturity T with period M in binomial models, the time until maturity $[0,T]$ partitioned into M sub-intervals of the same. The length of the partition, ie $\Delta t = \frac{T}{M}$. Let $t_i =$iΔt, showing partition points for $i = 0,1,2,\dots,$M, then for each sub-interval $(t_i, t_{i+}1]$,the asset price is assumed to be constant. If S_0 is the share price initially at time $t = 0$, and at $t = T$ will rise to opportunities p to S_0u or going down with odds of $1 - p$ to S_0d, where $u > 1$ and $1 > d > 0$ (Arum.Handini, 2013). Illustration Binomial models for 4 periods *(M= 4)* as in figure 1.
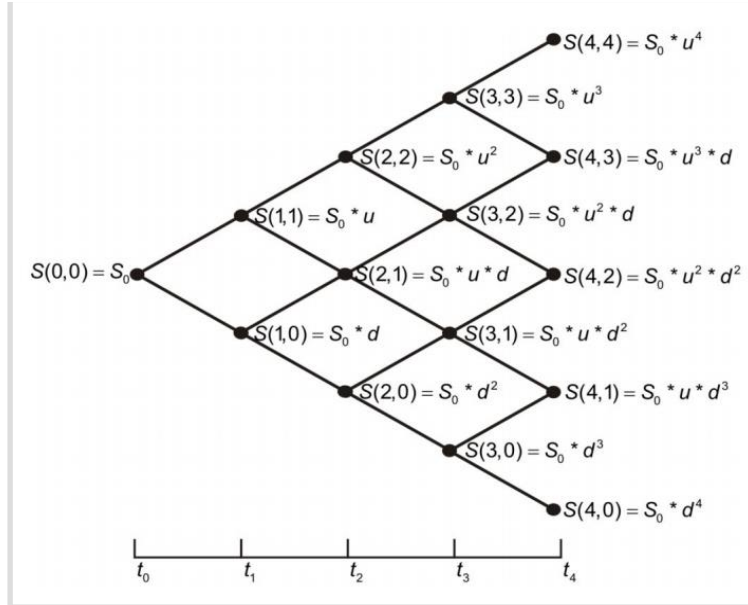
Figure 1 Illustration Binomial Tree Model to 4 periods

Description:

$u = e^{\sigma\sqrt{\Delta t}}, d = e^{-\sigma\sqrt{\Delta t}}$ and $p = \frac{(e^{r\Delta t} - d)}{ud,}$ where

σ = Volatility of stock

Δt = length of each period, $\Delta t = \frac{T}{M}$

M = Number of changes in stock prices within T years.

Option pricing model with binomial tree method performed by the Backward method. If known K is the price of an option contract and St is the price of the underlying asset at the time of maturity, then the gain callable at the time of maturity is the

$$f(S_T) = maks(S_T - K, 0),$$

Binomial Tree Model for Bermudan Option

Suppose Bermuda options with expiry T has a number N opportunity to exercise its option, ie $\{t_1, t_2, \ldots, t_n = T\} \subset [0, T]$, the final condition for Bermuda option equivalent to the final conditions on the American option. If K is the price of the option contract, $C_{Ber}S_t$ denotes the value of the Bermuda call option at time t, for, $0 \le t \le T$ at the price of assets S, the final condition for the Bermudan call option can be written as follows $C_{Ber}S_T payoff = S_T = (S_T - K)^+$

Like the American option value calculation, from the final condition $C_{ber}S_{T,}$, the calculation for the value of the options Bermuda with binomial tree method can be done with a backward method. The key is that we value the option at the end of the period for any possible share price. For $t \in \{T_1, t_2, \ldots, t\ N\}$, the value of $C_{Ber}S_t$, can not be less than the value of the payoff;

$$C_{Ber}(S_t) \ge (S_t - K)^+ \quad \text{for } t \in \{t_1, t_2, \ldots, t_N\},$$

Thus, the model binomial tree for calculation of callable type of Bermuda can be formulated as follows:

For each interval $[t_i, t_{i+1}) \subset [0, T], i = n$ - 1, n - 2, …, 1 and final condition

$C_{Ber}\left(S_{t_{i+1}}\right)$ calculation $C_{Ber}(S_t)$, t ϵ $[t_i, t_{i+1})$ completed with the following

$$\begin{cases} C_{Ber(S_{i-1})} = V(i,n) = maks\left[e^{-r\Delta t}(pV_{i+1,n+1}+(1-p)V_{i+1,n}),C(S_{i,n})\right],0 \le n \le i, \text{ for } t = t_i \\ C_{Ber(S_{i-1})} = V(i,n) = e^{-r\Delta t}(pV_{j+1,n+1}+(1-p)V_{i-1,n}),0 \le n \le i, \text{ for others } t \end{cases}$$

scheme:

wherein, $t_i < t_{i+1}|t_i, t_{i+1} \in \{t_1, t_2, \dots, t_N\}$.

Description:

S_i, n = stock price of
K = the option contract price
r = Free interest rate risks
Δt = length of each period ($\Delta t = T / m$)
T = Period Options
m = Period stock price change
$C(S_i, n)$ = Function advantage the call option, where:
$C(S_{i,n}) = max[S_{i,n} - K, 0], 0 \le n \le i, 0 \le i \le T - 1$

3. METHODOLOGY
The Place, Time and Type Data Research
This uses secondary data, ie historical data closing prices John Keells shares (JHK) taken on 09 October 2014 to October 9, 2015, and there are 239 in stock prices. Data obtained from one of the financial data providers in the city of Yogyakarta, the *BNI Financial Update* Corner, FEB. The data were taken in the form of stock prices, the price of the option *call* Bermuda type, price and options contracts maturing company of John Keels option.

Data Analysis Techniques
Data analysis in this research is through the method of tree Binomial (Binomial Tree Method) were performed using several steps, namely:

(1) Normality Test *Log Return* stocks using *Kolmogorov-Smirnov* to test whether a log *returns* stock come from populations with normal distribution or not. Furthermore, with P_t is the price of an asset in the period to t and P_{t-1} is the price of an asset in the period (*t-1*), the log *return* can be defined as follows:

$$R_t = Log\left(\frac{P_t}{P_{t-1}}\right)$$

(2) Estimates share price volatility determined using the historical volatility that aims to measure the level of risk of a stock. Furthermore, the historical volatility of the stock price can be calculated with the following formula:

$$\sigma = \sqrt{k \times \frac{\sum_{t=1}^{n}\left(R_t - \overline{R}_t\right)^2}{n-1}},$$

where : k = number of periods in a year trading

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

$$\overline{R}_t = \frac{1}{n}\sum_{t=1}^{n} R_t$$

(3)   The risk-free interest rate, the value of risk-free interest rate in this case study is taken from address: *the website* id.tradingeconomics.com

(4)   The calculation of the Bermudan-type call option using the binomial tree model (*binomial tree method*).

4.   DISCUSSION OF RESULTS

The stock data used in this study is the historical data on closing price John Keells (JHK). Stock data is taken from 09 October 2014 to October 9, 2015, and there are 239 in stock prices. The share price of John Keells (JHK) was taken on the share price on 20 October 2015 in the amount of 171.25 LKR. The above data will be used in calculating the price of the call option type of Bermudan through binomial tree method. The number of changes in stock prices *n* or the number of steps selected for the study was 20, 50, 80, 100, 200, 300, 400, 500, 600, 700, 800, 900 and 1000. The opportunities *exercise* or time allowed to run option until maturity of one month, the per 2 days, per 4 days and per week.

Historical data on stock prices will be used to calculate the log *returns* and the estimated historical volatility. Closing stock price movement chart of John Keells (JHK) for one year using Matlab R2010a program can be seen in figure (2).
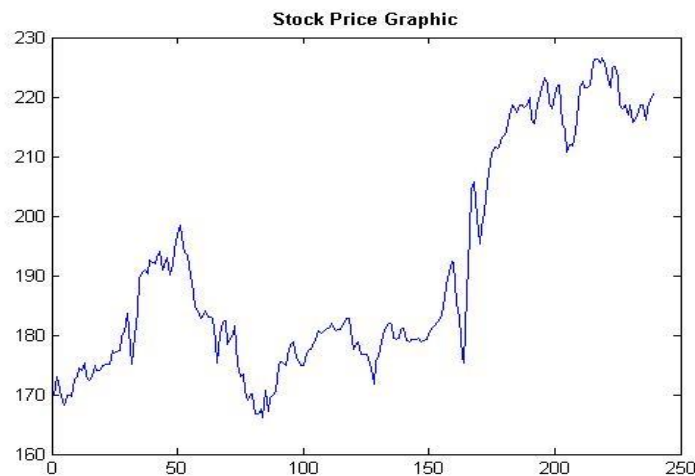

Figure 2 Plot John Keells Stock Price Movement Period October 9, 2014 - October 9, 2015
Source: *Output* program Matlab R2010a

Log Return Normality Test
Normality test was used to test the normality of *t*he log return stock of John Keells (JHK) is *Kolmogorov-Smirnov test.* Results of *output* SPSS 20. The results of the ou*tput* of SPSS 20 is as follows:

Table 1 Normal Test Logs *Return* Stock

One-Sample Kolmogorov-Smirnov Test

		Log_Return
N		209
Normal Parameters[a,b]	Mean	-,001487
	Std. Deviation	,0092747
Most Extreme Differences	Absolute	,071
	Positive	,071
	Negative	-,049
Kolmogorov-Smirnov Z		1,023
Asymp. Sig. (2-tailed)		,246

Source: *Output* Program SPSS 20

Based on output generated the above-obtained *p-value* for lognormality test return John Keells stock is 0.246. The *p-value* is greater than an *alpha* of 0.05. It can be concluded that the log return stock of John Keells (JHK) derived from a normally distributed population.

Volatility Estimation
Estimation results of historical volatility of John Keells shares (JHK) using Matlab R2010a program assistance were obtained for 0,199 or 19.9%.

Risk-Free Interest Rate
Risk-free interest rate value is taken from an address the website: *id.tradingeconomics.com.*The value of the risk-free interest rate is the interest rate in Sri-Lanka that is equal to 6%.

Pricing Call Option Bermudan Type
Using the information gained, the price of shares of John Keells (JHK) on 20 October 2015($S_{0)}$ is equal to 171.25 LKR, the contract price *(K)* for 161.87 LKR, risk-free interest rate (*r*) in Sri-Lanka is 6%, the volatility (σ) of 19.9% and maturity of the options is November 12, 2015. The purchase option price offered in the market that is equal to 9.47 LKR. The results of the comparison calculation of callable Bermudan type with purchase option European and American type using binomial models as follows:

Table 2 Option Price Comparison *Call* Bermuda, Europe and the United States John Keells shares, with the market price = 9.47 LKR

M	European Call Option	Bermudan Call Option			American Call Options
		per-2 days	**per-4 days**	**per week**	
20	10,8822	10,8822	10,8822	10,8822	10,8822
50	10,894	10,894	10,894	10,894	10,894
80	10,8903	10,8903	10,8903	10,8903	10,8903
100	10,8836	10,8836	10,8836	10,8836	10,8836
200	10,8844	10,8844	10,8844	10,8844	10,8844
300	10,8876	10,8876	10,8876	10,8876	10,8876
400	10,8864	10,8864	10,8864	10,8864	10,8864
500	10,8853	10,8853	10,8853	10,8853	10,8853
600	10,8852	10,8852	10,8852	10,8852	10,8852
700	10,8856	10,8856	10,8856	10,8856	10,8856
800	10,8862	10,8862	10,8862	10,8862	10,8862
900	10,8866	10,8866	10,8866	10,8866	10,8866
1000	**10,8867**	**10,8867**	**10,8867**	**10,8867**	**10,8867**

Source: Output data program Matlab R2010a

From the table 2, that for only one option and one option contract prices, it appears that the results of purchase option price calculation types Bermudan John Keells shares using CRR binomial method produce a value close to the price of purchase options on the market. The value of an option to European or American call purchase option is always greater than its intrinsic value, then the Bermudan call option will be equal to the value of European call option or American option call value. The value of call option itself looks fluctuate with the value of *M* is increasingly enlarged and will converge to a value for the value of *M* more. Below is a chart which shows the price movement of Bermudan call option.
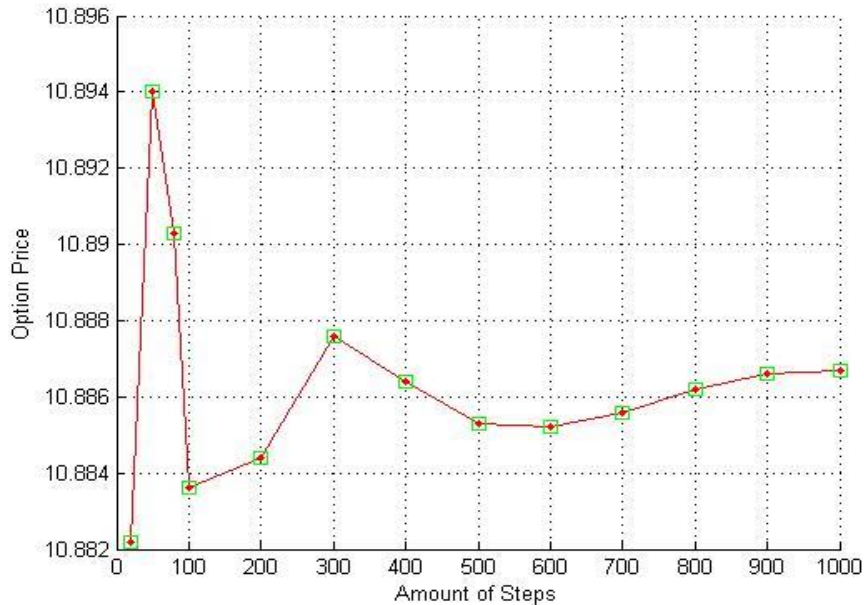
Figure 3 Options Price Movement Buy Bermuda Type Stocks John Keells
Sumber: *Output* program Matlab R2010a

5. CONCLUSIONS AND RECOMMENDATIONS
Conclusions
From the discussion that has been done, it can be concluded that:
1. The pricing of Bermuda options with binomial tree method is influenced by several factors, including the initial stock price of the initial stock price ($S_{0)}$, price contract (K),risk-free interest rate (r),volatility (σ),maturity (T), the number of changes in stock prices (M) and exercise opportunities.
2. Pricing Bermudan call option with the absence of a dividend payment using the binomial method, resulting in the same value when compared with European and American call options. Total number of occasions exercised the Bermudan call option does not affect changes in the value of the option.

Recommendations
From the discussion in this study, there are several things that can be suggested by the author for further research, among others:
1. Adding other parameters such as the level of dividends to determine how much influence the level of dividends to the value of Bermuda option.
2. Selection of alternative methods for calculating the price of options Bermuda is still possible to obtain more accurate results and approaching the market price.

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